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The supply of information by broadcast media

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1. Introduction

Although there can be hardly any doubt that broadcast media play a very important role in modern societies, economic analysis of their behaviour is a rare event. In many respects this is astonishing for the following reasons. Firstly, the relevant market is too large to be ignored. In the U.S.A., for example, about 98% of all households own a TV set and Nielsen (1983) estimates that, on average, people spend more than 50 hours per week watching TV programs. Secondly, during the last decade there has been a very controversial discussion especially in Europe whether or not private TV stations should be allowed. The antagonists of private broadcasting claimed that private networks would present too much poor quality entertainment and too little information. The protagonists answered that people always have the choice not to watch private programs and, therefore, private suppliers have to present exactly what the people want to see. A well-founded economic analysis of this problem is still pending. Certainly, with this study we hope to make a useful contribution to this discussion.

Our central interest is the information supply of broadcast media, especially TV networks. In economic theory, information is always modeled as an entity which people do not derive direct utility from, but which enables individuals to overcome uncertainty and therefore leads to 'better' consumption or investment decisions. In this sense information is not a consumption good. On the other hand, private networks produce 'news shows' to present

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information, and the name 'show' already indicates that they hope to entertain the viewer with this kind of program. We have to confess that information is neither a pure consumption good nor only used for the purpose of making better decisions. Although information seems to be something in between, we will concentrate our analysis on one end of the scale, namely the purely instrumental use of information. Private networks present news shows at prominent times in the program schedule, and this would hardly be the case if people only want to be entertained.

In view of the importance economic theory attaches to the phenomenon 'information', it is surprising that the literature does not provide any model that can be employed to analyse the information supply of mass media. In our opinion, the main difficulty in developing an adequate model lies in the necessity to cover heterogeneous preference for information. Mass media do not only supply a particular *amount* of information, but also information of a particular kind. Otherwise we could make use of information economic models like Kihlstrom (1974).

Recent publications, especially Allen (1986, 1990), present a framework which, in effect, allows the treatment of heterogeneous information demand. Unfortunately, these very general models are inappropriate to answer our particular questions. Actually, our approach turns out to be a special case of Allen's model. Furthermore, general information economic models attend the dependency of information demand and supply from information structures in *competitive* markets. In contrast, we need for our purpose a theory of information demand and a theory of information supply. While the former can easily be found in the literature, the latter is not available. There are two important reasons for this. Firstly, broadcasted information can be used without paying a price. Secondly, there are usually only a few suppliers. Therefore, an oligopolist market model would be adequate. As a first step and for the sake of simplicity we will, however, restrict the analysis to the case of only one supplier. Throughout this paper we describe the behavior of a profit-maximizing TV network with respect to its information supply, and investigate the relevant welfare effects. In our results we will show that the question, whether or not private information supply leads to welfare losses in comparison with 'public' supply, heavily depends on the opportunity costs to the viewers.

The paper is organized as follows: In section 2 we present a model, which can be seen as a special case of the general information model discussed in the relevant literature. This model serves as an *example* with the help of which we can demonstrate the central trade offs without too much technicalities. In section 3, we characterize the information value, analyse in 4 the profit-maximizing type of information, and in 5 the welfare-maximizing type. In section 6 we look at the private and the welfare maximizing choice of information amount and type and compare the private and public decisions.

In the last section we summarize our findings. All proofs and the evidence of differentiability and continuity are collected in an appendix.

2. The model

Throughout the model, information refers to states of nature which influence individual utility. S is the set of all possible states of nature and s is a single state. In the relevant literature, information is modeled as a sub-σ-field $\mathscr{F}$ of the given σ-field $\mathscr{S}$ on S.[1]

If an individual possesses the information $\mathscr{F}$, he is able to decide whether or not the event $F \in \mathscr{F}$ has occurred after one $s \in S$ has come to pass. For our purpose, we will use an example, which has a simpler structure than the general approach used in the information literature. As already mentioned in the introduction, information is seen as a means of enhancing the ability to make better decisions. For example, some TV stations provide information on financial markets. In this case the role of information is clear and the most general line of modelling information for such purposes is the one indicated above. However, the range of types of information covered by this approach is much broader. What is not covered here is the case, where information becomes consumption good per se. But this again was already mentioned in the introduction.

We define states of nature as elements of $S = [-\varepsilon, \varepsilon]$, where $\varepsilon \geqq 2$, and where we equate $-\varepsilon$ and ε, and therefore S is the circumference of a circle of length 2ε, as often used in the production differentiation literature. In this case $\mathscr{S}$ can be naturally defined as the Borel sets $\mathscr{B}$ on S. Without mass media, individuals have no information and they think of all states of nature as equally probable. Therefore, individuals' beliefs are modeled as the uniform distribution on S. In this respect individuals do not differ. Individually, utility depends on the consumption $x \in C$, and on the state of nature. The consumption space $C = \mathbb{R}^4_+$, where $(x_1, x_2, x_3, x_4) \in C$ can be interpreted as follows: (x_1, x_2) are consumption goods where we choose x_2 as numéraire, x_3 is the number of minutes people watch TV entertainment programs and x_4 are minutes of leisure time not spent in front of the TV set. Each consumer has the same endowment of R units of the numéraire good and X minutes of total leisure time. Hence, consumption without information is restricted to

$$C(0) = \{x \in C \,|\, px_1 + x_2 \leqq R, x_3 + x_4 \leqq X\}.$$

In this respect individuals do not differ either. The origin of heterogeneity of information preferences in our model lies in the assumption that different individuals are affected differently by the same state of nature and that they

[1]See, for example, Gilboa and Lehrer (1991).

have different opportunity cost of watching an information program. As to the latter type of differentiation of consumers we assume that there are two groups of consumers: the members of the first group spend their leisure time watching TV-entertainment (if provided), those of the second group choose different leisure activities. Both derive constant marginal utility k form one unit of time spent on their favourite activity. If the network supplies information and therefore cannot supply entertainment in the required period of time, the members of the first group only derive constant marginal utility of time $l \leqq k$ from a different leisure activity, if they do not watch the information program. A fraction, g, of all consumers belongs to the first group and a fraction, $1-g$, belongs to the second group.

As to the other type of differentiation of consumers, we choose the most simple nontrivial specification:

$$u_\beta(x_1, x_2) = b(\beta, s)x_1 + nx_2,$$

where $\beta, s \in S$ and β is consumer specific. Only the utility derived from the consumption of the first good is affected by the state of nature, and how concerned a consumer is depends on β. We assume that β is uniformly distributed on S. Hence there is a mass of 2ε consumers. We use the following specification:

$$b(\beta, s) := \max(0, a/2 - a\,|\beta - s|).$$

Hence the consumer derives the highest utility from x_1, if $s = \beta$. The more s differs from β, the less utility he derives from s. Put differently, the consumers are more affected by s close to β, while they are not affected by different s, which are distant from $\beta(|\beta - s| \geqq 0.5$ implies $b(\beta, s) = 0$!).

Summarizing, the total utility of consumers of the first group is

$$u_{\beta,\mathrm{TV}}(x_1, x_2, x_3, x_4, s) = b(\beta, s)x_1 + nx_2 + kx_3 + lx_4$$

while the total utility of the other group is

$$u_{\beta,\mathrm{F}}(x_1, x_2, x_3, x_4, s) = b(\beta, s)x_1 + nx_2 + lx_3 + kx_4.$$

Under these circumstances and if X time units of TV entertainment are supplied, the maximal utility of each consumer is $V_0 + kX$, where V_0 is the expected maximal utility derived from consumption (x_1, x_2). $V_0 = a/8\varepsilon \cdot R/p$ (see appendix). For our further analysis it will be very advantageous to use a special parameter constellation, where

$$\frac{a}{8\varepsilon}\,\frac{R}{p} = nR = 1.$$

This means that individuals are perfectly indifferent towards good 1 and good 2, as long as they can only rely on their own information. We will comment shortly on this assumption in section 3. The assumption $nR=1$ reduces the notational burden without restricting the generality of the results.

The network offers two kinds of goods, namely entertainment $z \in \mathbb{R}_+$ and information $\mathcal{G}$, which is a sub-σ-field of $\mathcal{S}$. We assume that it can offer either entertainment or information at any moment of time. For our example we assume that G takes the most simple nontrivial form: $[\alpha_1, \alpha_2] \subset S$.

$$\mathcal{G} = \{\{\}, S, [\alpha_1, \alpha_2], S \backslash [\alpha_1, \alpha_2]\}.$$

$\mathcal{G}$ can be generated by the random variable

$$\begin{aligned} Y(s) &= 1 \text{ for } s \in [\alpha_1, \alpha_2] \\ &= 0 \text{ for } s \in S \backslash [\alpha_1, \alpha_2]. \end{aligned}$$

This means that the network provides information about whether or not s lies in the interval $[\alpha_1, \alpha_2]$. While the structure of $\mathcal{G}$ is fixed, the length and the location of the interval $[\alpha_1, \alpha_2]$ can vary. If information is supplied, this takes $T=1$ units of time, where the assumption $T=1$ again does not restrict the generality of the results. The network then has two options: Either supply $z=X$ time units of entertainment and no information, or supply $z=X-1$ time units of entertainment and information characterized by Y.

Above, we have fully characterized the behaviour of the consumers if the network chooses the first option (no information). If the network chooses the second option (information), the consumers are affected as follows: Each consumer, who chooses to watch the information program, now has maximal utility $V_Y(\beta)+k(X-1)$ where V_Y is the expected maximal utility derived from consumption (x_1, x_2), given information Y. Consumers, who don't watch the information program, have maximal utility V_0+kX, if they belong to the second group, and $V_0+k(X-1)+l$, if they belong to the first group.

Therefore, an individual will demand information Y, iff

$$V_Y(\beta) - V_0 \geqq k \quad \text{for} \quad \text{F consumers,}$$

$$V_Y(\beta) - V_0 \geqq l \quad \text{for} \quad \text{TV consumers.}$$

The demand for information depends on the additional expected utility, which can be achieved by using Y:

$$v^*(\beta, \alpha) := V_Y(\beta) - V_0,$$

where $\alpha=|\alpha_2-\alpha_1|$.[2] This additional utility depends on the relative positions of $[\beta-1/2,\beta+1/2]$ and $[\alpha_1,\alpha_2]$. With its choice of α the networks present a partition of S which generates signals, the value of which depends on the individual preference of each consumer.

Besides $v^*(\beta,\alpha)$, information demand also depends on the opportunity costs of information processing. Consumers can use leisure time to process information, or to watch TV entertainment programs or for activities in which they do not use the TV set at all. Opportunity costs of information processing therefore consist of the lost utility from TV entertainment or from other leisure activities. *If* information is presented, the two groups (TV consumers, F consumers) are differently affected. TV consumers now only derive utility l during $T=1$. Therefore, the number of TV consumers demanding Y is

$$\mu(\alpha,l):=\lambda(\beta\,|\,v^*(\beta,\alpha)\geqq l),$$

where λ is the Lebesgue measure on S. F consumers do not lose leisure time options. Therefore, the number of F consumers demanding Y is

$$\mu(\alpha,k):=\lambda(\beta\,|\,v^*(\beta,\alpha)\leqq k).$$

Hence the total number of spectators of Y is

$$(1-g)\mu(\alpha,k)+g\mu(\alpha,l).$$

Here we assume that for each β there are g TV consumers and $1-g$ F consumers. If information is supplied, the number of 'viewer-minutes' the network gets is

$$(1-g)\mu(\alpha,k)+g\mu(\alpha,l)+g2\varepsilon(X-1),$$

while no information supply leads to

$$g2\varepsilon X$$

'viewer-minutes'.

Having described the demand side of the model, we now turn to the supply side. We assume that the network maximizes the (advertising-)

[2]Given that consumers are uniformly distributed on S with respect to β, it is easy to check that the location of $[\alpha_1,\alpha_2]$ influences neither the return of the network nor the general welfare. Therefore α alone characterizes the pay-off relevant impact of Y. In what follows, Y is identified by α and we assume $(\alpha_1+\alpha_2)/2=0$.

proceeds, which are proportional to the 'viewer-minutes' as defined above. Furthermore, production costs are constant (fixed) costs.

The network has to decide (1) whether to supply information or not and (2), if it supplies information, it has to choose the profit maximizing α. (1) relates to the *amount* of information, (2) relates to the *precision* of information to which we turn in the following section.

Given that the network wants to maximize the total number of viewer-minutes, information Y will be supplied if

$$\max_{\alpha}\,(1-g)\mu(\alpha,k)+g\mu(\alpha,l)\geqq g2\varepsilon. \tag{1}$$

It should be emphasized, that the expression 'viewer minutes' does not denote the time *one* consumer spends in front of the TV-set but the total time spent by *all* individuals which can be reached by the supplied program.

Note also that consumers are perfectly indifferent with respect to the particular time slot, $T=1$, within X. One can view the decision (1) of the network as a decision, whether to use one time slot for an informative program or not. As the network only provides one piece of information, Y, at most, only one time slot is relevant for this decision. Within each time slot the network maximizes its audience.

For any further analysis we need to know more about $v^*(\beta,\alpha)$. Therefore we will first take a closer look at the demand for information of type Y. After that we will analyse the profit maximizing choice of α and then determine the welfare maximizing α. The last step will be the analysis of the welfare and the profit maximizing decision about both, the amount *and* type of information.

Obviously the chosen example is of a structure which, on the one hand, allows an adequate analysis of our problem and, on the other hand, minimizes technicalities.

3. The value of information

As long as individuals rely only on their own information, they are confronted with a lottery which assigns equal probability to each possible state of nature (fig. 1).

The signal $Y(s)=i$ $(i=1,0)$ offered by the network can be interpreted as two alternative lotteries (figs. 2 and 3).

With probability $\alpha/2\varepsilon$ the consumer finds himself after the receipt of $Y(s)$ in a situation described by fig. 2, and with probability $(2\varepsilon-\alpha)/2\varepsilon$ he is confronted with lottery II. Both lotteries may be 'good' or 'bad' news. They are 'good' news if $E(b(\beta,s)\,|\,Y(s)=y)>E(b(\beta,s))$ and 'bad' news if $E(b(\beta,s)\,|\,Y(s)=y))\leqq E(b,(\beta,s))$. This qualification obviously refers to the marginal utility derived from the consumption of good 1. If we refer to the total utility, there can be no 'bad' news at all because the certain consump-

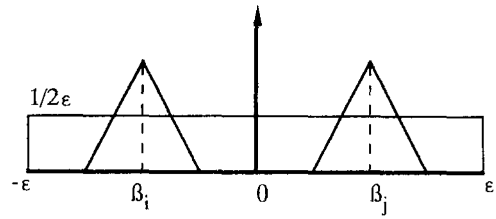

Fig. 1

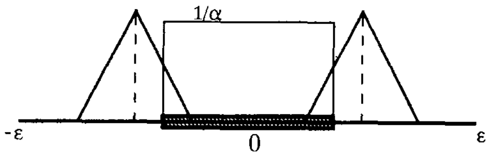

Fig. 2. Lottery I.

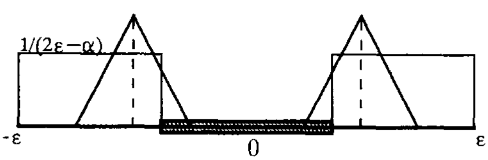

Fig. 3. Lottery II.

tion of $nR = 1$ units of good 2 is always available: If the consumer receives 'good' news, he will use his whole endowment for good 1, and if he unfortunately receives 'bad' news, the whole endowment is used to consume x_2. There could be 'bad' news about the expected utility if we were to allow $E(b(\beta, s)) = aR((8\varepsilon p) > nR$, but this would lead to an additional case without gaining any further insights and, because we have a great number of cases anyway, we leave this out. As a consequence of this assumption the value of information depends only on the effect of 'good' news and the probability of its occurrence.

The value information has for a particular consumer depends on the relative positions of the intervals $[\beta - 1/2, \beta + 1/2]$ and $[\alpha_1, \alpha_2]$. For a β located close to the middlepoint of the interval $[\alpha_1, \alpha_2]$ the value of information is high because the good news facilitates a good discrimination between high and low levels of the marginal utility $b(\beta, s)$. For these β's receiving $Y = 1$ means 'good' news. The more we move to the edge of the interval, the more the ability to discriminate decreases up to a critical point where neither $Y = 1$ nor $Y = 0$ is good news. At this point the signal discriminates as badly as the private information between high and low levels of $b(\beta, s)$. For higher β's $Y = 0$ becomes good news and the ability to discriminate increases. Beginning from $\beta \geqq (\alpha+1)/2$ all positive utilities lie in the support of $Y=0$ and, therefore, nothing changes in the ability to discriminate the signals. Hence, because of non-homogeneous preferences for information it should be clear that we will not find that a special degree of precision will have the same value for all individuals. A consumer with $\beta=0$ will tend to desire a small α because, from his point of view, such a signal allows a better discrimination of 'good' and 'bad' news. On the other hand, an individual with, for example $\beta \notin [\alpha_1, \alpha_2]$ will have different wishes on $(\alpha_2-\alpha_1)$; in fact, such an individual will prefer a greater α. Therefore, 'precision' has only a plain meaning if we define it with respect to a single individual. This fact is also clearly reflected in the graphs of v^*, which can be found in the appendix.

On the basis of this knowledge together with the formal description of $v^*(\beta, \alpha)$ developed in the appendix we are now able to analyse the profit-maximizing type of information.

4. The profit-maximizing type of information

In this section we will analyse the type of information a private network supplies if it supplies non-trivial information. In section 2 we have already shown that in order to determine the profit maximizing type of information, $\alpha_m(k,l)$, the problem

$$\max_{\alpha}\ (1-g)\mu(\alpha,k)+g\mu(\alpha,l) \tag{2}$$

has to be solved. First, we will look exclusively at the two extreme cases where $k=l>0$ and $k>l=0$. For $k=l>0$, (2) reduces to $\mu(\alpha,k)$. For $l=0$ is $\mu(\alpha,0)=2\varepsilon$ so that in both cases only $\mu(\alpha,k)$ counts. In both cases we get $\alpha_m(k,k)=\alpha_m(k,0)=:\alpha_m(k)$ as a solution of

$$\max_{\alpha}\ \mu(\alpha,k). \tag{3}$$

As already indicated by the notation, the number of consumers who watch information programs heavily depends on k. At high enough opportunity costs no individual is willing to watch these programs. Because of

$$\max_{\beta,\alpha}\ v^*(\beta,\alpha)=\left(\frac{4\varepsilon-1}{4\varepsilon}\right)^2,$$

which is shown in the appendix, we can conclude:

Proposition 1. *For $k \geqq (4\varepsilon-1/4\varepsilon)^2$ and all $\alpha \in (0,2\varepsilon]$. it is true that $\mu(\alpha,k)=0$.*

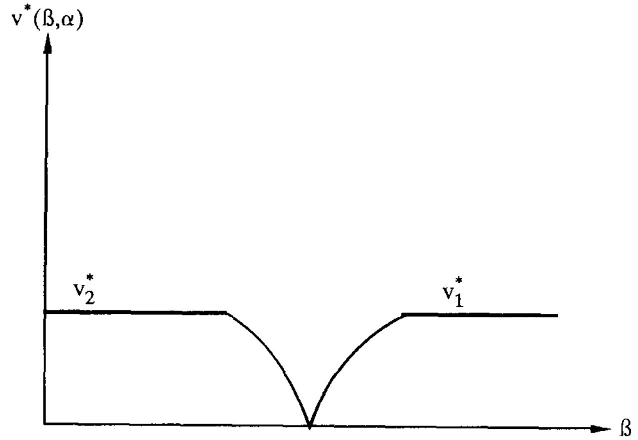

Fig. 4

It follows from the symmetry of our example that we can concentrate on the case $\alpha \leqq \varepsilon$. For $\alpha = \varepsilon$ we get $v_1^* = v_2^* = 0.5$ (fig. 4), where v_2^* is the value to consumers with $[\beta - 1/2, \beta + 1/2] \subset [\alpha_1, \alpha_2]$ ($Y(s) = 1$ is good news) and v_1^* is analogously defined (see appendix).

For $\alpha < \varepsilon$ we get $v_1^* < 0.5 < v_2^*$ (see appendix). Consequently, for $k > 0.5$, only those individuals can be reached by an information program, for whom $Y = 1$ is 'good' news. The supplier will therefore make his choice in order to fulfil the desire of this group. More exactly the following holds:

Proposition 2. For

$$\frac{1}{2} < k \leqq \frac{2\varepsilon - 1}{2\varepsilon}$$

the following holds:

$$\alpha_m(k, k) = 2\varepsilon(1 - k) - \frac{1}{4\varepsilon}; \quad \mu_m(k) = 2\varepsilon(1 - k) - \frac{4\varepsilon - 1}{4\varepsilon}.$$

For

$$\frac{2\varepsilon - 1}{2\varepsilon} \leqq k \leqq \left(\frac{4\varepsilon - 1}{4\varepsilon}\right)^2 \text{ the following holds:}$$

$$\alpha_m(k, k) = \frac{4\varepsilon - 1}{4\varepsilon}; \quad \mu_m(k) = \left(\left(\frac{4\varepsilon - 1}{4\varepsilon}\right)^2 - k\right)^{1/2}.$$

It remains for us to check, which type of information the network will supply in the case of low opportunity-costs, $k \leqq 1/2$. In this case, the supplier has three options. He can choose α in such a way that $v_1^*(\beta,\alpha) \geqq k$ and $v_2^*(\beta,\alpha) < k$, or so that $v_1^*(\beta,\alpha) < k$ and $v_2^*(\beta,\alpha) \geqq k$ and finally so that v_1^* as well as v_2^* is greater or equal to k. It can be shown (see appendix) that the first two options can never lead to a profit-maximizing type of information. Under these conditions the following proposition follows immediately:

Proposition 3. If $k \leqq 1/2$, the profit maximizing type of information is $\alpha_m(k) = \varepsilon$ and

$$\mu_m(k) = 2\varepsilon - 2 + 2(1-2k)^{1/2}.$$

Thereby the profit-maximizing type of information and the corresponding numbers of viewers are fully described for the two extreme cases $l = k$ and $l = 0$. Not surprisingly it turns out that the higher the opportunity-costs k are, the more the network orientates itself on the desires of those individuals next to 0 (for $\beta = 0$, $\alpha = 1 - 1/4\varepsilon$ maximizes the information value).

In an earlier version of the paper, we also considered the intermediate cases $0 < l < k$. While technically involved, such an analysis sheds no further important insights with respect to our central question.

The essential finding is quite in line with intuition: the higher opportunity cost k, the harder it is to get viewers for an information program. Those who watch are those with β close to 0. Therefore the higher k the more α is tuned to the needs of this group. Note that for the extreme cases considered $l = 0$ and $l = k$, the needs of the individuals who give priority to entertainment are not considered separately at all. For $l = 0$, this group watches anyway and for $k = l$ the incentive to provide a particular α is the same for both groups.

Of course, up to now we have made no statement about the question of whether or not Y will at all be supplied. Before we turn to this question, we will first analyse the welfare-maximizing type of information.

5. The welfare-maximizing type of information

In contrast to a private supplier, for a social planner not only is the number of viewers of importance, but also the additional utility individuals derive from information.

Let $B(\alpha,k) := \{\beta \mid v^*(\beta,\alpha) \geqq k\}$ be the set of those consumers of information programs who have only low interest in TV entertainment. The total utility of those individuals who do not like to spend their leisure time watching TV entertainment then is

$$\int_{B(\alpha,k)} V_Y(\beta)\,\mathrm{d}\beta + (2\varepsilon - \mu(\alpha,k))\,V_0 + (2\varepsilon - \mu(\alpha,k))k + 2\varepsilon k(X-1)$$

$$= \int_{B(\alpha,k)} v^*(\beta,\alpha)\,\mathrm{d}\beta + 2\varepsilon(V_0 + k(X-1)) + (2\varepsilon - \mu(\alpha,k))k$$

and the total utility of individuals preferring TV entertainment is

$$\int_{B(\alpha,l)} v^*(\beta,\alpha)\,\mathrm{d}\beta + 2\varepsilon(V_0 + k(X-1)) + (2\varepsilon - \mu(\alpha,l))l.$$

If we denote

$$V(\alpha,k) := \int_{B(\alpha,k)} v^*(\beta,\alpha)\,\mathrm{d}\beta - \mu(\alpha,k)k \tag{4}$$

the welfare-maximizing type of information is a solution of

$$\max_{\alpha}\ (1-g)V(\alpha,k) + gV(\alpha,l), \tag{5}$$

which we denote by $\alpha_w(k,l)$. Note that in contrast to the private supplier case $\alpha_w(k,k) \neq \alpha_w(k,0)$ in general.
Again, we first analyze the case $k=l$, i.e. the problem

$$\max_{\alpha}\ V(\alpha,k). \tag{6}$$

As we have seen in section 4, when

$$k \geqq \left(\frac{4\varepsilon-1}{4\varepsilon}\right)^2$$

nobody watches information programs, and we therefore can restrict our analysis to the cases in which

$$k < \left(\frac{4\varepsilon-1}{4\varepsilon}\right)^2.$$

In section 4 we have also shown that for $k > 1/2$ only those individuals are reached for whom $Y=1$ is good news. In this case, the following holds:

Proposition 4. For

$$k \geqq \frac{2\varepsilon-1}{2\varepsilon},$$

we have

$$\alpha_w(k,k)=1-\frac{1}{4\varepsilon}.$$

This is not surprising because the private network gets the greatest number of viewers for these parameter values if it offers the individuals with β close to 0 their most preferred type of information, $\alpha_m(k,k)=1-1/4\varepsilon$. Therefore, these individuals consume information from which they derive maximal utility and consequently the aggregated additional utility from information is also maximal.

Proposition 5. For

$$\frac{1}{2}<k<\frac{2\varepsilon-1}{2\varepsilon},$$

$\alpha_w(k,k)$ is defined as the solution of

$$\varepsilon-\alpha+\frac{1}{2}-\frac{1}{4\varepsilon}-\frac{1}{2}\left[\frac{1}{\varepsilon}\left(\alpha-1+\frac{1}{4\varepsilon}\right)\right]^{1/2}=\varepsilon k$$

and we have

$$\frac{\partial\alpha_w(k,k)}{\partial k}<0 \quad \textit{and} \quad \frac{\partial^2\alpha_w(k,k)}{\partial k^2}>0$$

and for

$$k=\frac{2\varepsilon-1}{2\varepsilon}: \alpha_w(k,k)=1-\frac{1}{4\varepsilon}.$$

For $k\leqq 1/2$ the analysis is more difficult than in the private case because for these parameters $V(\alpha,k)$ is not quasi-concave in general. This follows from the fact that for the social planner, the decision whether to reach individuals for whom $Y=1$ as well as $Y=0$ is good news, or not, is not as easy as for the private supplier. The private network always prefers the first alternative because it reaches twice as many viewers. If, for example, $k=1/2$, the private information type is $\alpha_m(k)=\varepsilon$. But in this case the additional utility is 0, whereas Proposition 5 tells us that the additional utility for $\alpha_w(k)$ will be positive. In general, for $k\leqq 1/2$ we always have to compare the local maximum of the group for which both $Y=1$ and $Y=0$ are good news with the local maximum of the group for which only $Y=1$ is a good news. The appropriate analysis leads to:

Proposition 6. There exists a ϱ, $1/3<\varrho<1/2$, *such that for* $\varrho \leqq k \leqq 1/2$, $\alpha_w(k)$ *as in Proposition 5 and for* $0 \leqq k \leqq \varrho$, $\alpha_w(k)=\varepsilon$.

Hence, as in the case of a private supplier, the information is more tuned to the needs of the consumers with β close to zero the higher are the opportunity cost k.

Let us now turn to the case $l=0$. For $k<\varrho$, it is trivial that $\alpha_w(k,0)=\varepsilon$ and for $\varrho<k$, we get

$$\alpha_w(k,k) \leqq \alpha_w(k,0) \leqq \alpha_w(0,0)=\varepsilon.$$

Qualitatively these results are similar to those obtained for the private supplier. But there is one essential difference: the private network only considers the number of viewers but not the utility generated by the information program, which is the primary concern of a social planner. If $l=k$, this has the consequence that a social planner will choose in general a smaller α, because this increases the utility of the viewers of the information program at the cost of losing some viewers:

Proposition 7.

$$\alpha_w(k,k) \leqq \alpha_m(k) \quad \textit{and} \quad \alpha_w(k,k)<\alpha_m(k) \quad \textit{iff} \quad \varrho<k<\frac{2\varepsilon-1}{2\varepsilon}.$$

This relation remains the same, if l differs only a little from k. But for large differences the relation can be reversed. For $l=0$ and $k=(2\varepsilon-1)/2\varepsilon$, $\alpha_m(k)=\alpha_w(k,k)=1-1/4\varepsilon$. But for $\alpha<\varepsilon$, $\partial V(k,0)/\partial\alpha>0$ and therefore $\alpha_w(k,0)>\alpha_w(k,k)$. Hence, in this case $\alpha_w(k,0)>\alpha_m(k,0)$. It is again easy to see the reason for this. If $l=0$, a private supplier does not have to take into account the desires of the $Y=0$ individuals. Moreover from a social point of view the value that information has for these individuals has to be considered if we are to decide about the type of information. And this group of individuals is best served by $\alpha=\varepsilon>1-1/\varepsilon$. We should also note here that the analysis of the extreme cases can be extended to the case $0<l<k$ without additional insights.

6. The total decision

Up to now we have only analyzed what *kind* of information will be supplied, given that information has been supplied anyway. In this section we will deal with the question of under what circumstances a positive amount of information will be supplied at all.

For the private network, we have already seen in section 2 that for $l=k$ and $l=0$ supplying information is profit-maximizing exactly if

$$(1-g)\mu(\alpha_m(k),k)+g\mu(\alpha_m(k),l)\geqq g2\varepsilon. \tag{7}$$

It follows immediately that, for $l=0$, information will always be supplied. ($\mu(\alpha_m(k),0)=2\varepsilon$). Individuals who derive nearly no utility from leisure activities apart from TV entertainment, will watch the information program in any case if they are kept from looking at entertainment programs. Hence, the network does not lose any viewers if information is presented; on the contrary, it finds new customers.

For the other extreme case $l=k$, (7) reduces to

$$\mu(\alpha_m(k),k)\geqq g2\varepsilon.$$

For sufficiently small g and sufficiently small k, information supply will always maximize the profit. Putting both cases together we get:

Proposition 8. There exists for all k a $g_m(k)$ and for all $g<1$ a $k_m(g)$ such that information supply is profit maximizing
(a) if $l=0$
(b) if $l=k$ and: $g\leqq g_m(k)$ or $k\leqq k_m(g)$.

Let us now turn to the decision of the welfare-maximizing social planner. In section 5 we have already seen that the total utility of information supply is

$$(1-g)V(\alpha,k)+gV(\alpha,l)+2\varepsilon(V_0+kX)-g2\varepsilon(k-l).$$

Without information supply the total utility is $2\varepsilon(V_0+kX)$. Hence, information supply is welfare-maximizing exactly if

$$(1-g)\,V(\alpha_w(k,l),k)+gV(\alpha_w(k,l),l)\geqq g2\varepsilon(k-l).$$

It follows immediately that, for $l=k$ information supply always leads to a welfare gain. This is easy to see because if $l=k$, nobody suffers a loss of utility due to the information supply. On the other hand, if $l=0$, information will not be supplied in every case, but only if

$$(1-g)\,V(\alpha_w(k,0),k)+gV(\alpha_w(k,0),0)\geqq g2\varepsilon k.$$

If g approaches 1, $\alpha_w(k,0)$ tends to ε. But because for $k>1/2$ $v^*(\beta,\varepsilon)<k$ always holds, it follows that information supply causes a welfare loss. If, on the other hand, g is close to 0, information supply is socially meaningful. As

$$\frac{\partial}{\partial g}\left[(1-g)\,V(\alpha_w(k,0),k)+gV(\alpha_w(k,0),0)-g2\varepsilon(k)\right]<0$$

we can prove:

Proposition 9. There exists for all k a $g_w(k)$ and for all g a $k_w(g)$ such that information supply is welfare-maximizing
(a) if $l=k$, or
(b) if $l=0$ and $g \leqq g_w(k)$ or $k \leqq k_w(g)$.

Again, these results can be extended to the intermediate cases $0<l<k$. The fact that a high value of $l(l=k)$ leads to the provision of information from the social point of view, while in the private case this result is induced by a low value of $l(l=0)$ can then be shown to take the form, that the private incentive to provide information decreases in l, while the social one increases in l.

7. Summary

Comparing the results for the private and the public networks, it turns out that decreasing g and decreasing k in both cases make information supply more desirable, while changes in l lead to opposite reactions of private and public suppliers. It is mainly the latter point, namely the different influence of l, which prevents us from making general statements like 'private networks never offer enough information'. Not only is it possible for a private supplier to provide too *little* information (if $l=k$), it is also possible for it to provide too *much* (if $l=0$). Furthermore, private and social optimisation can lead to the same amount and the same type of information (if $k<\varrho$). For all other parameter constellations, the results of private and public reasoning are different, at least with regard to the type of information.

Summarizing, we can conclude that it is not possible to state in general that private networks supply too little and/or too unprecise information. Rather, how efficiently private networks act, heavily depends on the opportunity costs l and k and the proportion of TV consumers g. Furthermore, it turns out that in view of the heterogeneity of information preferences, 'precision' cannot be defined as an objective term, independent of personal preferences, and, therefore, to claim 'objective' precise information is obviously meaningless! In some situations, private networks also look at the desires of the viewers when deciding what kind of information is to be supplied, because otherwise these viewers cannot be reached. If we think of our analysis with $l=k$ and k close to $(2\varepsilon-1)/2\varepsilon$, this implies that a private network will tend to ignore the desires of its target audience (β close to 0) in order to get more viewers, although this may reduce the total additional utility of information.

The complain that private networks present only inadequate information programs can, with the help of our example, be supported for small $k-l$ and $l>\varrho$; but it has to be rejected for small l (close to 0) and sufficiently high k. In the latter case, private networks supply information that serves their target group (for which $Y=1$ is good news) very well (especially better than α_w if it were supplied). And this is the case even if from a social point of view the supply of information is not justifiable.

As already emphasized, our model serves as an *example* which we use as a simple means to demonstrate the relevant trade-offs networks are confronted with when to decide about amount and type of their information supply. We believe that this example may be a good starting point for a more comprehensive and/or more general analysis of media markets. We further hope, that the model can also be used in order to analyse the dissemination of information in other cases as the one we discussed here. As one of our referees stated it, the model could be a 'potentially useful way of thinking about informative advertising.' We agree with this and believe that there may be even more fields at which the model can be fruitfully employed. As examples think of political advertising or information behaviour of bureaucrats.

Appendix

This appendix gathers the main arguments necessary to prove the assertions of the paper. Some details, however, are omitted. An extended appendix containing a fully detailed discussion is available upon request from the authors.

A.1. Properties of $v^*(\beta,\alpha)$

Simple calculation yields

$$E(b(\beta,s))=\int_{-\varepsilon}^{\varepsilon} b(\beta,s)\,\mathrm{d}s=\frac{a}{8\varepsilon}$$

which implies $V_0=a/8\varepsilon\ R/p=nR=1$, where the latter equation derives from the assumptions $a=8\varepsilon pn$.

The symmetry of the model implies $v^*(\beta,\alpha)=v^*(-\beta,\alpha)$. Therefore we consider only $0\leqq\beta\leqq\varepsilon$. To derive the functional form of v^* we have to distinguish several cases.

We start with $\beta\geqq(\alpha+1)/2$. Hence we analyze individuals with support $(b(\beta,s))\subset\{s\,|\,Y(s)=0\}$.

We find

$$E(b(\beta,s)\,|\,Y=0)=\frac{2\varepsilon}{2\varepsilon-\alpha}\,\frac{a}{8\varepsilon}$$

and

$$E(b(\beta,s)\,|\,Y=1)=0.$$

As

$$\frac{2\varepsilon}{2\varepsilon-\alpha}\geqq 1,$$

maximal utility for

$$Y=0 \text{ is } \frac{2\varepsilon}{2\varepsilon-\alpha},$$

while it is 1 for $Y=1$. Hence

$$V_Y(\beta)=\frac{\alpha}{2\varepsilon}+\frac{2\varepsilon-\alpha}{2\varepsilon}\,\frac{2\varepsilon}{2\varepsilon-\alpha}=\frac{\alpha}{2\varepsilon}+1$$

and $v^*(\beta,\alpha)=V_Y(\beta)-V_0=\alpha/2\varepsilon$.

Hence, summarizing:

1) $$v^*(\beta,\alpha)=v_1^*(\beta,\alpha):=\frac{\alpha}{2\varepsilon} \quad \text{for} \quad \frac{\alpha+1}{2}\leqq\beta\leqq 1.$$

Analogous reasoning yields for $\sigma_1:=\beta-\alpha/2$ and $\sigma:=-\sigma_1$.

2) $$v^*(\beta,\alpha)=v_2^*(\beta,\alpha):=\frac{2\varepsilon-\alpha}{2\varepsilon} \quad \text{for} \quad 0\leqq\beta\leqq\frac{\alpha-1}{2}.$$

3a) $$v^*(\beta,\alpha)=v_3^{*-}(\beta,\alpha):=\left(\frac{\varepsilon-\alpha}{2\varepsilon}-2\sigma_1(1-\sigma_1)\right)$$

$$\text{for } \max\left(\frac{\alpha}{2},\frac{1-\alpha}{2}\right)\leqq\beta\leqq\frac{\alpha+1}{2} \quad \text{and} \quad \frac{\varepsilon-\alpha}{2\varepsilon}\geqq 2\sigma_1(1-\sigma_1)$$

$v_3^{*-}(\beta,\alpha)$ is decreasing and convex in β.

3b) $$v^*(\beta,\alpha)=v_3^{*+}(\beta,\alpha):=-v_3^{*-}(\beta,\alpha)$$

$$\text{for } \max\left(\frac{1-\alpha}{2},\frac{\alpha}{2}\right)\leqq\beta\leqq\frac{\alpha+1}{2} \quad\text{and}\quad \frac{\varepsilon-\alpha}{2\varepsilon}\leqq 2\sigma_1(1-\sigma_1)$$

$v_3^{*-}(\beta,\alpha)$ is increasing and concave in β.

4) $$v^*(\beta,\alpha)=v_4^*(\beta,\alpha):=\left(\frac{\varepsilon-\alpha}{2\varepsilon}+2\sigma(1-\sigma)\right)$$

for

$$\left|\frac{\alpha-1}{2}\right|\leqq\beta\leqq\frac{\alpha}{2},$$

$v_4^*(\beta,\alpha)$ is decreasing and concave in β.

5) $$v^*(\beta,\alpha)=v_5^*(\beta,\alpha):=(2\alpha-\alpha^2-4\beta^2-\alpha/4\varepsilon)$$

for $0\leqq\beta\leqq\min((1-\alpha)/2,\alpha/2)$

$v_5^*(\beta,\alpha)$ is decreasing and concave in β.

6a) $$v^*(\beta,\alpha)=v_6^{*-}(\beta,\alpha):=\frac{\alpha}{2\varepsilon}\,(4\varepsilon(1-2\beta)-1)$$

for $\frac{\alpha}{2}\leqq\beta\leqq\frac{1-\alpha}{2}$ and $4\varepsilon(1-2\beta)\geqq 1$.

6b) $$v^*(\beta,\alpha)=v_6^{*+}(\beta,\alpha):=-v_6^{*-}(\beta,\alpha)$$

for $\frac{\alpha}{2}\leqq\beta\leqq\frac{1-\alpha}{2}$ and $4\varepsilon(1-2\beta)\leqq 1$.

It is easy to check.

Lemma 1. *$v^*(\beta,\alpha)$ is continuous on $[0,\varepsilon]^2$ and continuously differentiable, where $v^*(\beta,\alpha)>0$.*

The structure of $v^*(\cdot,\alpha)$ is summarized in the following figs. A.1 to A.4. Going through tedious calculations yields:

Lemma 2. *For all $0\leqq\alpha\leqq\varepsilon$, $v^*(0,\alpha)=\max_{0\leqq\beta\leqq\varepsilon}v^*(\beta,\alpha)$.*

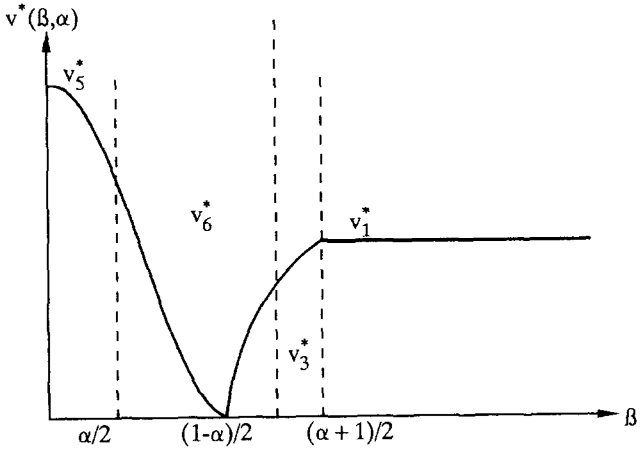

Fig. A.1. $0<\alpha<1/4\varepsilon$.

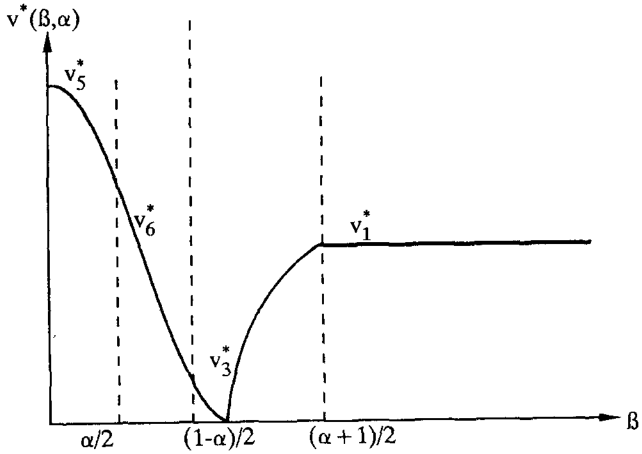

Fig. A.2. $1/4\varepsilon<\alpha<1/2$.

Lemma 3. $\alpha=1-1/(4\varepsilon)$ *solves* $\max_{0\leqq\alpha\leqq\varepsilon} v_5^*(\beta,\alpha)$ *for all* $\beta\geqq 0$ *and*

$$\max_{\substack{0\leqq\alpha\leqq\varepsilon\\0\leqq\beta\leqq\varepsilon}} v^*(\beta,\alpha)=\max_{0\leqq\alpha\leqq\varepsilon} v_5^*(0,\alpha)=\left(\frac{4\varepsilon-1}{4\varepsilon}\right)^2.$$

A.2. Properties of $\mu(\alpha,k)$

By definition $\mu(\alpha,k):=\mu(B(\alpha,k))$ with

$$B(\alpha,k):=\{\beta\,|\,v^*(\beta,\alpha)\geqq k\}.$$

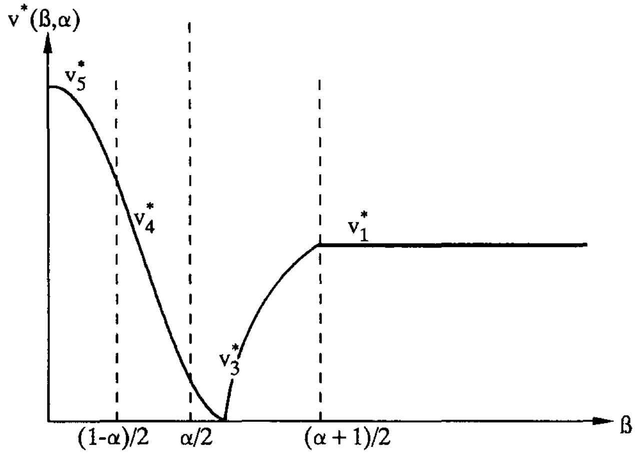

Fig. A.3. $1/2<\alpha<1$.

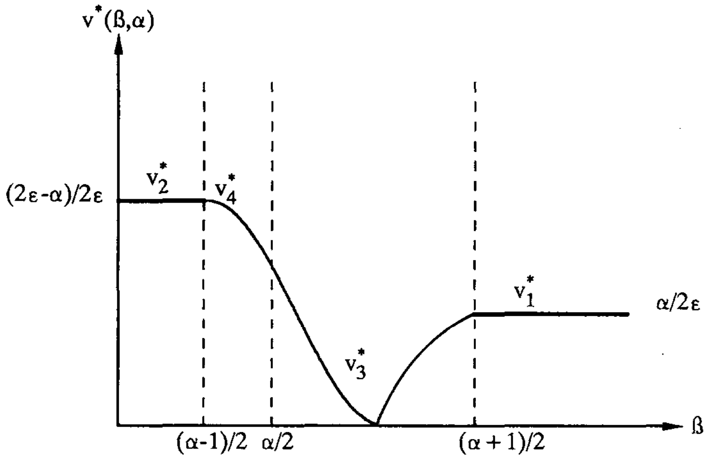

Fig. A.4. $1<\alpha<\varepsilon$.

Because of the symmetry of the model, we have

$$\mu(\alpha,k)=2\mu(B^+(\alpha,k))=:2\mu^+(\alpha,k) \quad \text{with} \quad B^+(\alpha,k)=B(\alpha,k)\cap\mathbb{R}_+.$$

Using the monotonicity properties of $v^*(\cdot,\alpha)$ it is meaningful to define $\beta^-(\alpha,k)$ as the solution of $v^*(\beta,\alpha)=k$ in the decreasing part of $v^*(\cdot,\alpha)$ with the convention that $\beta^-(\alpha,k)=0$, if no such solution exists, and equal to the maximum of the solutions, if several solutions exist (only relevant for $v_2^*(\beta,\alpha)=k$).

Analogously let $\beta^+(\alpha,k)$ the solution of the above equation in the increasing part of $v^*(\cdot,\alpha)$ with the convention that $\beta^+(\alpha,k)=\varepsilon$, if no such solution exists, and equal to the smallest solution, if several solutions exist $(v_1^*(\beta,\alpha)=k)$.

Then

$$B^+(\alpha,k)=\{\beta\geqq 0\,|\,\beta\leqq\beta^-(\alpha,k)\quad\text{or}\quad\beta\geqq\beta^+(\alpha,k)\}$$

and therefore

$$\mu^+(\alpha,k)=\beta^-(\alpha,k)+\varepsilon-\beta^+(\alpha,k).$$

Let $A^-(k):=\{\alpha\leqq\varepsilon\,|\max(v_5^*(0,\alpha),\ v_2^*(0,\alpha))\geqq k\}$ be the largest closed set, such that $\alpha\in\operatorname{int}(A^-(k))$ implies $\beta^-(\alpha,k)>0$. Analogously, let $A^+(k):=\{\alpha\in A^-(k)\,|\, v_1^*(\varepsilon,\alpha)\geqq k\}$ be the largest closed set, such that $\alpha\in\operatorname{int}(A^+(k))$ implies $\beta^+(\alpha,k)<\varepsilon$.

It is easy but tedious to check

Lemma 4. $\beta^+(\alpha,k)$ *is differentiable on* $\{(\alpha,k)\,|\,\alpha\in\operatorname{int}A^+(k)\}$ *and* $\beta^-(\alpha,k)$ *is differentiable on* $\{(\alpha,k)\,|\,\alpha\in\operatorname{int}A^-(k)\}$.

Hence

$$\mu^+(\alpha,k)=\beta^-(\alpha,k)\quad\text{for}\quad\alpha\in A^-(k)\backslash A^+(k),$$

$$\mu^+(\alpha,k)=\beta^-(\alpha,k)+\varepsilon-\beta^+(\alpha,k)\quad\text{for}\quad\alpha\in A^+(k)$$

and $\mu^+(\alpha,k)$ is differentiable on $\{(\alpha,k)\,|\,\alpha\in\operatorname{int}A^+(k)\}$ and on $\{(\alpha,k)\,|\,\alpha\in\operatorname{int}(A^-(k)\backslash A^+(k))\}$.

For each k we can now determine the solutions to

$$\max_{\alpha}\ \mu^+(\alpha,k)\quad\text{st}\quad\alpha\in\operatorname{cl}(A^-(k)\backslash A^+(k))\tag{A.1}$$

and

$$\max_{\alpha}\ \mu^+(\alpha,k)\quad\text{st}\quad\alpha\in A^+(k).\tag{A.2}$$

It is easy to see that μ^+ can be continuously extended to the closure in (A.1). As a final step in order to solve

$$\max_{\alpha}\ \mu^+(\alpha,k)\quad\text{st}\quad\alpha\in A^-(k)\tag{A.3}$$

we have then to compare the solutions to (A.1) and (A.2) and to select the one with the larger value of μ^+.

Unfortunately, this program turns out to be technically quite involved for two reasons: As the functional form of $\beta^{+-}(\alpha, k)$ depend on the functional form of v^*, we first have to determine the set of (α, k), where each of the versions of v^* is relevant for $\beta^{+-}(\alpha, k)$. Secondly, $\beta^{+-}(\alpha, k)$ turn out to be not concave in general. Therefore, we have to check for quasiconcavity in order to get a unique solution to (A.1) and (A.2) respectively. The result of such an analysis is that

(a) the maximizing α is such that

$$\alpha \geqq 1 - \frac{1}{4\varepsilon}.$$

(b) $\mu^+(\cdot, k)$ is concave for such

$$\alpha \geqq 1 - \frac{1}{4\varepsilon}$$

in $A^+(k)$ and $A^-(k) \backslash A^+(k)$ respectively.

(c) in a neighborhood around the maximizing α, β^- is determined by

$$v_5^* \quad \text{for} \quad k > \frac{2\varepsilon - 1}{2\varepsilon},$$

$$v_4^* \quad \text{for} \quad k < \frac{2\varepsilon - 1}{2\varepsilon},$$

and β^+ is determined by v_3^{*+}.

Around the maximizing α this implies the functional form

$$\beta^-(\alpha, k) = \beta_5(\alpha, k) := \frac{1}{2}\left(2\alpha - \alpha^2 - \frac{\alpha}{2\varepsilon} - k\right)^{1/2} \quad \text{for} \quad k > \frac{2\varepsilon - 1}{2\varepsilon}$$

$$= \beta_4(\alpha, k) := \frac{\alpha - 1}{2} + \left(\frac{1}{2} - \frac{k}{2} - \frac{\alpha}{4\varepsilon}\right)^{1/2} \quad \text{for} \quad k < \frac{2\varepsilon - 1}{2\varepsilon},$$

$$\beta^+(\alpha, k) = \beta_3^+(\alpha, k) := \frac{\alpha + 1}{2} - \left(\frac{\alpha}{4\varepsilon} - \frac{k}{2}\right)^{1/2}.$$

Using these functional forms and comparing the solutions of (A.1) and (A.2) gives for the solution $\alpha_m(k)$ to (A.3):

Lemma 5.

$$For\ \frac{2\varepsilon-1}{2\varepsilon} \leqq k \leqq \left(\frac{4\varepsilon-1}{4\varepsilon}\right)^2: \quad \alpha_m(k)=\frac{4\varepsilon-1}{4\varepsilon}.$$

$$For\ \frac{1}{2}<k\leqq\frac{2\varepsilon-1}{2\varepsilon}: \alpha_m(k)=2\varepsilon(1-k)-\frac{1}{4\varepsilon}.$$

$$For\ 0\leqq k\leqq\tfrac{1}{2}: \alpha_m(k)=\varepsilon.$$

And for $\mu_m(k):=2\mu^+(\alpha_m(k),k)$ we obtain by inserting

Lemma 6.

$$\mu_m(k)=2\varepsilon-2+2(1-2k)^{1/2} \quad for \quad 0\leqq k\leqq\tfrac{1}{2},$$

$$\mu_m(k)=2\varepsilon(1-k)-\left(\frac{4\varepsilon-1}{4\varepsilon}\right) \quad for \quad \frac{1}{2}<k\leqq\frac{2\varepsilon-1}{2\varepsilon},$$

$$\mu_m(k)=\left(\left(\frac{4\varepsilon-1}{4\varepsilon}\right)^2-k\right) \quad for \quad \frac{2\varepsilon-1}{2\varepsilon}\leqq k\leqq\left(\frac{4\varepsilon-1}{4\varepsilon}\right)^2,$$

$$\mu_m(k)=0 \quad for \left(\frac{4\varepsilon-1}{4\varepsilon}\right)^2\leqq k.$$

A.3. Properties of $V(\alpha,k)$

By definition

$$V(\alpha,k)=\int_{B(\alpha,k)} v^*(\beta,\alpha)\,\mathrm{d}\beta-\mu(\alpha,k)k=\int_{B(\alpha,k)}(v^*(\beta,\alpha)-k)\,\mathrm{d}\beta.$$

Because of the symmetry of the model, we have

$$V(\alpha,k)=2V^+(\alpha,k)=2\int_{\beta^+(\alpha,k)}(v^*(\beta,\alpha)-k)\,\mathrm{d}\beta.$$

Therefore, we shall analyze $V^+(\alpha,k)$ instead. Because of the arguments in *A.2*, we know

$$V^+(\alpha,k)=\int_0^{\beta^-(\alpha,k)}(v^*(\beta,\alpha)-k)\,\mathrm{d}\beta+\int_{\beta^+(\alpha,k)}^{\varepsilon}(v^*(\beta,\alpha)-k)\,\mathrm{d}\beta.$$

In order to find the solution, $\alpha_w(k)$, to

$$\max_{\alpha \in A^-(k)} V(\alpha, k) \tag{A.4}$$

we proceed analogously to section A.2. Note that such a solution exists, as $V(\cdot, k)$ is continuous even at $\alpha = 2\varepsilon\, k (\Leftrightarrow v_1^*(\varepsilon, \alpha) = k)$. This is so, because

$$\int_{\beta^+}^{\varepsilon} (v^*(\beta, \alpha) - k)\,\mathrm{d}\beta = 0 \quad \text{for} \quad \text{such } \alpha.$$

$V^+(\alpha, k)$ inherits the differentiability properties of β^+ and β^-. The problems of finding the maximizing α in $\mathrm{cl}(A^-(k) \setminus A^+(k))$ and in $A^+(k)$, which were discussed in section A.2 with respect to $\mu^+(\alpha, k)$ are also present here. Moreover, we find that for $1/2 - 1/16\varepsilon^2 < k < 1/2 - 1/32\varepsilon^2$, $V^+(\alpha, k)$ is not even quasiconcave in $A^+(k)$. Rather $V^+(\alpha, k)$ is quasiconvex on these sets. Analyzing the resulting complications yields the following structure:

(a) there is a $\varrho \in]1/3, 1/2[$ such that there is a unique solution $\alpha_w(k) \geqq 1 - 1/4\varepsilon$ for $k \neq \varrho$.
(b) $V^+(\cdot, k)$ is quasiconcave on $A^-(k)$ for $k \leqq 1/2 - 1/16\varepsilon^2$ and $k \geqq 1/2 - 1/32\varepsilon^2$.
(c) in a neighbourhood of $\alpha_w(k)$,

$$\beta^-(\alpha, k) = \beta_5(\alpha, k) \quad \text{for} \quad k > \frac{2\varepsilon - 1}{2\varepsilon},$$

$$\beta^-(\alpha, k) = \beta_4(\alpha, k) \quad \text{for} \quad k > \frac{2\varepsilon - 1}{2\varepsilon},$$

$$\beta^+(\alpha, k) = \beta_3^+(\alpha, k).$$

Using these functional forms we find

Lemma 7. There exists a $\varrho \in]1/3, 1/2[$ such that

$$\textit{for} \quad \frac{2\varepsilon - 1}{2\varepsilon} \leqq k \leqq \left(\frac{4\varepsilon - 1}{4\varepsilon}\right)^2, \quad \alpha_w(k) = \frac{4\varepsilon - 1}{4\varepsilon} = \alpha_m(k)$$

$$\textit{for} \quad \varrho < k < \frac{2\varepsilon - 1}{2\varepsilon}, \quad \alpha_w(k) < \alpha_m(k) \quad \textit{and} \quad \alpha_w(k) \textit{ solves}$$

$$\varepsilon - \alpha + \frac{1}{2} - \frac{1}{4\varepsilon} - \frac{1}{2}\left[\frac{1}{\varepsilon}\left(\alpha - 1 + \frac{1}{4\varepsilon}\right)\right]^{1/2} = \varepsilon k$$

for $0 \leqq k < \varrho$, $\alpha_w(k) = \varepsilon = \alpha_m(k)$.

Lemma 8. *For $\varrho < k < (2\varepsilon - 1)/2\varepsilon$, $\alpha(\cdot)$ is strictly decreasing and strictly convex.*

Lemma 9.

$$\frac{\partial V^+}{\partial k}(\alpha_w(k), k) = -\mu(\alpha_w(k), k) < 0 \quad \textit{for} \quad k \neq \varrho.$$

A.4. Propositions

Most of the propositions are direct consequences of the lemmata of the preceding sections.

Proposition 1 follows from Lemma 3; Propositions 2 and 3 are contained in Lemmata 5 and 6.

Now consider

$$\max_{0 \leqq \alpha \leqq \varepsilon} \mu(\alpha, k).$$

This problem has a solution because of the continuity of $\mu(\cdot, k)$ on the subsets discussed in A.2. Denote the solution by $\alpha_m(k)$.

Propositions 4, 5 and 6 are contained in Lemmata 7 and 8.

Now consider

$$\max_{0 \leqq \alpha \leqq \varepsilon} (1-g)\; V^+(\alpha, k) + gV^+(\alpha, l).$$

This problem has a solution due to the continuity of $V^+(\cdot, k)$. Denote its solution by $\alpha_w(k, l)$.

Proposition 7 is contained in Lemma 7.

Consider for

$$k < \left(\frac{4\varepsilon - 1}{4\varepsilon}\right)^2$$

$$(1-g)\mu(\alpha_m(k), k) + g(\mu(\alpha_m(k), l) - 2\varepsilon) =: F_m(k, l, g).$$

Observe that $F_m(k, l, 0) > 0 \geqq F_m(k, l, 1)$.

$$F_m(k, 0, g) > 0 \quad \text{for} \quad g < 1,$$

where differentiable, F_m has the following derivatives:

$$\frac{\partial F_m}{\partial g} = \mu(\alpha_m(k), l) - 2\varepsilon - \mu(\alpha_m(k), k) \leqq 0,$$

$$\frac{\partial F_m}{\partial k} = (1-g)\,\frac{\partial \mu}{\partial k}\,(\alpha_m(k), k) \leqq 0.$$

Proposition 8 follows directly from the monotonicity properties of F_m. Finally consider for

$$k < \left(\frac{4\varepsilon - 1}{4\varepsilon}\right)^2$$

$$F_w(k, l, g) := (1-g)\,V(\alpha_w(\cdot), k) + g V(\alpha_w(\cdot)), l) - g2\varepsilon(k-l).$$

Clearly $F_w(k, l, 0) > 0$, $F_w(k, k, g) > 0$.

Where defined, we have

$$\frac{\partial F_w}{\partial g} = V(\alpha_w(\cdot), l) - V(\alpha_w(\cdot), k) - 2\varepsilon(k-l) \leqq 0,$$

$$\frac{\partial F_w}{\partial k} = (1-g)\,\frac{\partial V}{\partial k}\,(\alpha_w(\cdot), k) - g2\varepsilon < 0.$$

Proposition 9 follows directly from the monotonicity properties of F_w.

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